Stable multiderivative time-integrators for differential equations

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Short Description

Multiderivative (MD) time-integrators are methods that solve differential equations of the form

$$F(w, \partial_t w, ..., \partial_t^r w) = 0$$

for w by actively incorporating derivatives $\partial_t^k w\,(k \in \{1, ..., r\})$ into the numerical scheme. For example, in the case of initial value problems

$$w'(t) = f(w(t)),$$

an MD time-integrator at least computes values for the second derivative

$$w''(t) = \frac{\partial f(w)}{\partial w} f(w),$$

and possibly even higher derivatives orders. This entails that the Jacobian $\frac{\partial f(w)}{\partial w}$ is demanded in software implementations either exactly or through an approximation. Doing so incorporates more differential information at each timestep, and thus allows more easily for the development of high-order schemes. For ordinary differential equations, where $w(t)$ only depends on t, such methods have been theoretically studied already as early as the 1940's. Nevertheless, they have not been put much into practice, likely due to the demanding formulas that arise for the derivatives $\partial_t^k w$. In the context of partial differential equations, where $w(t, x)$ additionally depends on other variables $x = (x_1, ..., x_n)$, matters become even more cumbersome. Depending on the relation $F = F(w, \partial_t w, ..., \partial_t^r w)$, which can be very complicated, the inclusion of derivatives $\partial_t^k w$ needs to be treated with extra care as there might be x-derivatives involved. In the context of hyperbolic conservation laws, considerable effort has been put into applying the Cauchy-Kovalevskaya procedure as a workaround to transform x-derivatives into the requested t-derivatives.

During this mini-symposium we aim at highlighting the recent developments that have been made towards stable MD methods which are accessible as a viable alternative for more established methods. By cause of increasing hardware capabilities we can now reap the benefits of methods that years ago were practically infeasible. Now, being able to consider schemes that before were deemed inefficient, has led to the construction of MD methods with stability properties that standard multistep or multistage schemes can not have. A variety of MD methods will be presented ranging from MD compact implicit schemes to deal with timestep restrictions and stiffness, to novel MD multistage and MD general linear methods that take both efficiency and stability into account.